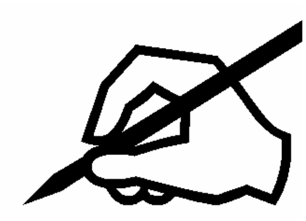


LESSON SUMMARY

CXC CSEC MATHEMATICS

Lesson 7

UNIT Five:
Algebra

Manipulating Algebraic Expressions, Equations and Inequalities

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1

(Some helpful exercises and page numbers are given throughout the lesson, e.g. Ex 6b page 232)

INTRODUCTION

The ability to manipulate algebraic expressions and equations is a very important skill in Mathematics. Many real life problems can be solved using algebra. The basic ideas in simplifying algebraic expressions and solving equations and inequalities will be developed in this lesson.

OBJECTIVES

At the end of this lesson you will be able to:

a) perform the four basic operations with algebraic expressions
b) substitute numbers for algebraic symbols
c) use symbols to represent binary operations and perform simple computations with them
d) simplify algebraic expressions
e) solve linear equations in one unknown
f) solve a simple linear inequality in one unknown
g) use linear equation to solve word problems

5.1 Definition of a variable

Letters or symbols used in mathematical expressions to stand for different numbers are called variables. If the value of a letter or symbol does not change it is called a constant.

Example: Identify the variables and constants in the following expression:

The formula for circumference of a circle is given by $C = 2\pi r$

Solution: the variables are C and r since these value can change in the expression. The constants are 2 and π since these values are fixed.

5.2 Evaluating algebraic expressions

This involves substituting numbers for the variables in the expression. In doing so a numerical expression is obtained.

Example:

If $x = 2$, $y = 3$ and $z = 4$, calculate the value of each of the following algebraic expression:

$24 \div xyz$

$24 \div 2 \times 3 \times 4$

$24 \div 24$

$= 1$.

ACTIVITY 1

Given that $a = 2, b = 3$ and $c = 5,$ determine the value of the algebraic expression (Ex 6b page 232):

$\frac{3a^4}{c^2} - 6a$.

5.3 Simplifying algebraic expressions

Remember only like terms can be added or subtracted. You may group them to perform these operations.

Example:

Simplify the following algebraic expressions: (Ex 6c page 234)

$9x + 3y - 5z + 8y - 4x + 12z$.

Solution:

$9x - 4x + 8y + 3y + 12z - 5z$

$5x + 11y + 7z$.

You can multiply or divide unlike terms. Remember that commutative and distributive laws hold for algebraic terms also. (Revise these laws now if you are unsure of them)

Simplify the following algebraic expressions: (Ex 6e page239)

$\frac{1}{3}(6x - 1) - \frac{1}{6}(12x - 3)$

Solution: The $- \times -$ gave a $+$.

$2x - \frac{1}{3} - 2x + \frac{1}{2}$

$= 2x - 2x + \frac{1}{2} - \frac{1}{3}$

$= 0 + \frac{1}{6}$

$= \frac{1}{6}$

Simplifying algebraic expressions with symbols that represent binary operations

A binary operation combines two numbers to form a third number. These may include operations other than the four basic operations.

Example: An operation is defined by $a * b = 3a - b$.
Calculate the exact value of $2 * 3$. (Ex 6f page 242)

Solution:

The operation requires that you multiply the first number by 3 and subtract the second number.

Therefore $2 * 3 = 3 \times 2 - 3$

$= 6 - 3$

$= 3.$

Simplifying Algebraic Fractions

Addition and subtraction of Algebraic Fractions

Example: Simplify the following (Ex 6k page250):

$\frac{8x}{9y} + \frac{2y}{3x}$

Solution:

Find the L. C. M as usual:

3	$9y$	$3x$
3	$3y$	x
x	y	x
y	y	1
	1	1

$\text{L.C.M} = 3 \times 3 \times x \times y$

The L.C.M of $9y$ and $3x$ is therefore $9xy$

$$\frac{8x}{9y} + \frac{2y}{3x}$$

$$\frac{8x^2 + 6y^2}{9xy}$$

$\frac{9xy}{3x} = 3y$

$3y \times 2y = 6y^2$

Try to do the working for $8x^2$

Multiplication and Division of Algebraic Fractions

Simplify the following (Ex 6l page 251)

$$\frac{3a^2}{4y} \div \frac{6a}{2y^2}$$

Solution:

When you divide by a fraction invert and multiply.

$$\frac{3a^2}{4y} \times \frac{2y^2}{6a}$$

$$\frac{\cancel{3}\cancel{a^2}^{a}}{\cancel{4}_2\cancel{y}} \times \frac{2\cancel{y^2}^{y}}{\cancel{6}_2\cancel{a}} = \frac{a}{2} \times \frac{y}{2}$$

$$\frac{ay}{4}.$$

ACTIVITY 2

1. Simplify the following algebraic expressions:

i) $6a^2b + 3ab^2 - 2a^2b - 4ab^2 + 3a^2b^2$.

ii) $\frac{3\,(x-5)}{7} - \frac{4\,(x-1)}{3}$

2. An operation is defined by $p \circ q = p^2 + q^3$.

State the exact value of $5 \circ (-4)$.

5.4 Solving simple equations in one variable

To solve an equation for an unknown variable you have to transpose the equation for the variable. Use the reverse of the operations involved to transpose.

Examples:

Solve each of the following equation (Ex 6m page 255):

1. $6x + 3 = 15$

Solution:

$6x + 3 - 3 = 15 - 3$

$6x = 12$

$\frac{6x}{6} = \frac{12}{6}$

$x = 2$.

2. $\sqrt{\frac{y}{2} - 3} = 5$

Solution:

Square both sides of the equation.

$$\left(\sqrt{\frac{y}{2}-3}\right)^2 = 5^2$$

$$\frac{y}{2} - 3 = 25$$

$$\frac{y}{2} - 3 + 3 = 25 + 3$$

$$\frac{y}{2} = 28$$

$$\frac{y}{2} \times 2 = 28 \times 2$$

$$y = 56$$

It is important to be able to translate word problems into algebraic symbols. Look at the following example.

Example:

Kelly had 12 dollars and spent x dollars. Ami had 6 dollars and collected x dollars. The two girls then had the same amount of money. Form an equation and solve it to determine the value of x. (Ex 6p page39)

Solution:

Kelly has $12 - x$ dollars.

Ami has $6 + x$ dollars.

They both have the same amount of money therefore:

$$12 - x = 6 + x$$

Add x to both sides of the equation.

$$12 - x + x = 6 + x + x$$

$12 = 6 + 2x$

Subtract 6 from both sides of the equation.

$12 - 6 = 6 - 6 + 2x$

$6 = 2x$

Divide both sides by 2.

$\frac{6}{2} = \frac{2x}{2}$

$3 = x$

Or

$x = \$3.$

ACTIVITY 3

1. Solve the following equation:

   $x - 3 = \frac{2x + 3}{5}.$

2. The length of a rectangle is 5 cm more than its width. If its perimeter is 58 cm, calculate its dimensions.

5.5 Solving simple inequalities and inequations in one variable

The steps in solving inequations are similar to solving equations. Just remember to flip the inequation sign when you multiply or divide by a negative number.

Example:

Solve the following inequation state the solution set and represent it on a graph. (Ex 6n page 261)

$3x + 2 \leq 5x - 4$

Solution:

Minus 2 from both sides

$3x + 2 - 2 \leq 5x - 4 - 2$

$3x \leq 5x - 6$

Minus $5x$ from both sides

$3x - 5x \leq 5x - 5x - 6$
$-2x \leq -6$

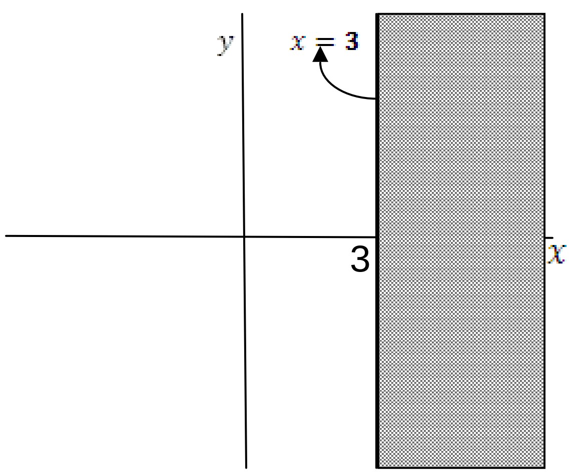
Divide both sides by -2 remember to flip the sign

$\frac{-2x}{-2} \geq \frac{-6}{-2}$

$x \geq 3$.

Solution set: $\{x: x \geq 3\}$

Solution on a graph locate the line $x = 3$. The solution is the region to the right of this line (shaded region):



A solid line was used because $x = 3$ is part of the solution. If it was not part of the solution a dashed line would have been used.

Some word problems can be expressed as inequations and solved.

Example:

The area of a rectangle must not be more than $198\ cm^2$. If the length of the length of the rectangle is $18\ cm$, calculate the greatest possible value of its width. (Ex 6q page 271)

Solution:

The area of a rectangle is given by:

$A = lb$ therefore,

$18cm \times b \leq 198\ cm^2$

$$b \leq \frac{198\ cm^2}{18cm}$$

$b \leq 11cm.$

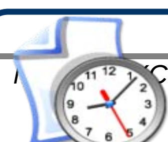
The greatest possible value of the width is $11cm.$

ACTIVITY 4

1. Solve the following inequations:

   $\frac{2}{9}x + \frac{5}{2} > \frac{1}{3}x - \frac{1}{2}$.

2. The area of a triangle must not be more than $102\ cm^2$. If the length of the base of the triangle is $12\ cm$, calculate the greatest possible value of its altitude.



ASSESSMENT

(Past CXC Questions)

1. (a) Simplify

(i) $3m - 2(m+1)$

(ii) $\frac{3}{y} - \frac{2}{y-2}$

(b) Solve the following equation

$2(x-1) = \frac{5}{2}$.

(c) Calculate the range of values of v when $5 - v \leq 2v - 1$.

CONCLUSION

In this lesson we looked at simplifying algebraic expressions and solving equations and inequalities. In lesson eight we will look at Factorization. This is also very important in simplifying expressions and will assist in solving equations.